Math III Review Rational and Radical Equations

Simplifying Rational and Radical Equations

Ex/ Solve $2x = \sqrt{5x - 1} + 1$

    Step 1: Subtract 1 from each side to isolate the radical term

$$2x - 1 = \sqrt{5x - 1}$$

    Step 2: Square both sides

$$4x^2 - 4x + 1 = 5x - 1$$

    Step 3: Set the right side equal to zero

$$4x^2 - 9x + 2 = 0$$

    Step 4: Solve for x (factoring, quadratic formula, graphing)

$$x = \frac{1}{4} \quad and \quad x = 2$$

    Step 5: Plug answers back into the original equation and check for extraneous solutions

$$2\left(\tfrac{1}{4}\right) = \sqrt{5\left(\tfrac{1}{4}\right) - 1} + 1 \qquad\qquad 2(2) = \sqrt{5(2) - 1} + 1$$

$$\tfrac{1}{2} \neq 1\tfrac{1}{2} \qquad\qquad\qquad 4 = 4$$

    So $x = \frac{1}{4}$ is **not** a solution   So x = 2 is a solution

The solution $\frac{1}{4}$ is an **extraneous solution** because is a solution to the transformed equation, not to the original equation

Ex/ Solve $\frac{x}{x-1} - 1 = \frac{x}{2}$

Step 1: Get a common denominator, in this case $2(x-1)$

$$\frac{2x}{2(x-1)} - \frac{2(x-1)}{2(x-1)} = \frac{x(x-1)}{2(x-1)}$$

Step 2: Since the denominators are the same we only need to simplify the numerator

$$2x - 2(x-1) = x(x-1)$$
$$2x - 2x + 2 = x^2 - x$$
$$0 = x^2 - x - 2$$

Step 3: Solve for x

$$0 = (x-2)(x+1)$$

So x = 2 *and* x = -1

Step 4: Plug answers back into the original equation and check for extraneous solutions

$$\frac{2}{2-1} - 1 = \frac{2}{2} \qquad\qquad \frac{-1}{-1-1} - 1 = \frac{-1}{1}$$
$$1 = 1 \qquad\qquad\qquad -\frac{1}{2} \neq -1$$

So x = 2 is a solution   So x = -1 is **not** a solution

The solution -1 is an **extraneous solution**

Examples:

1. Solve for x: $\frac{x+1}{5} - 2 = \frac{-4}{x}$
    A. x = 4
    B. x = 5
    C. x = 4, 5
    D. no solution

2. Solve for x: $\dfrac{8}{x-5} - \dfrac{9}{x-4} = \dfrac{5}{x^2-9x+20}$

3. Solve for x: $8 - \sqrt{x+12} = 3$

4. Solve for x: $\sqrt{x+15} = 5 + \sqrt{x}$