Math III Review

Polynomials A.APR.1, A.APR.2, A.APR.3, A.REI.4b, F.IF.7b, F.IF.8, N.CN.7

Adding polynomials – Add like terms, the exponents don't change!

Ex/ $(3x^2 - 4 + 2x) + (5x - 6x^2 + 7) = -3x^2 + 7x + 3$

Subtracting polynomials – Keep the first polynomial the same, change the subtraction to addition, and change the signs of the second polynomial. Exponents don't change.

Ex/ $(3x^2 - 4 + 2x) - (5x - 6x^2 + 7) = 9x^2 - 3x - 11$

Multiplying Polynomials – Each term in a polynomial has to be multiplied to each term in the other polynomial. Exponents change when terms are multiplied.

Ex/ $(2x^2 - 6x + 1)(x + 3)$

$2x^3 + 6x^2 - 6x^2 - 18x + x + 3$

$2x^3 - 17x + 3$

Ex/ $(x + 5)(x - 2)(3x + 4)$

$x^2 - 2x + 5x - 10\ (3x + 4)$

$(x^2 + 3x - 10)(3x + 4)$

$3x^3 + 4x^2 + 9x^2 + 12x - 30x - 40$

$3x^3 + 13x^2 - 18x - 40$

Dividing Polynomials – Can divide using synthetic or long division

Synthetic

$(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$

$$\begin{array}{r|rrrr} 4 & 2 & -13 & 26 & -24 \\ & & 8 & -20 & 24 \\ \hline & 2 & -5 & 6 & 0 \end{array}$$

Long Division

$(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$

$$\begin{array}{r} 2x^2 - 5x + 6 \\ x - 4\overline{)2x^3 - 13x^2 + 26x - 24} \\ \underline{(-)\ 2x^3 - 8x^2} \\ -5x^2 + 26x \\ \underline{(-)\ -5x^2 + 20x} \\ 6x - 24 \\ \underline{(-)\ 6x - 24} \\ 0 \end{array}$$

Roots, Zeroes, X-Intercepts – Are all solutions to polynomials

Finding the polynomial given the roots

Ex/ Find a 3rd degree polynomial given the roots 2 and $3i$

$(x-2)(x-3i)(x+3i)$

$(x-2)(x^2+3xi-3xi-9i^2)$

$(x-2)(x^2+9)$

$x^3-2x^2+9x-18$

Ex/ Find the roots of $x^3-4x^2+4x-16$

-Graph the polynomial and find a zero. This polynomial crosses the x-axis at 4.

-Divide $(x-4)$ from the polynomial.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 4 & -16 \\ & & 4 & 0 & 16 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

-So the polynomial is reduced to x^2+4. Can use either QF or solve the square root equation.

$x^2+4=0$

$-4 \quad -4$

$x^2=-4$

$\sqrt{x^2}=\pm\sqrt{-4}$

$x=\pm 2i$

So the roots of the polynomial are 4, $2i$, $-2i$

Review Examples

1. Which expression is equivalent to $(x+3)^3-9x(x+3)$?
   A. x^3+27
   B. x^3-27
   C. $x^3-9x^2-27x+27$
   D. $x^3-9x^2+27x+27$

2. The volume of a rectangular prism is represented by the expression $(x^3 - 2x^2 - 20x - 24)$. If the length is $(x - 6)$ and the height and width are equal, what is the width of the prism?
   A. $x + 2$
   B. $x - 2$
   C. $x + 4$
   D. $x - 4$

3. Suppose $p(x) = x^3 - 2x^2 + 13x + k$. The remainder of the division of $p(x)$ by $(x + 1)$ is -8. What is the remainder of the division of $p(x)$ by $(x - 1)$?
   A. -8
   B. 8
   C. 16
   D. 20

4. Which expression is the factored form of $x^3 + 2x^2 - 5x - 6$?
   A. $(x + 1)(x + 1)(x - 6)$
   B. $(x + 2)(2x - 5)(x - 6)$
   C. $(x + 3)(x + 1)(x - 2)$
   D. $(x - 3)(x - 1)(x + 2)$

5. What are the zeroes of the polynomial function $y = 2x^3 - 7x^2 + 2x + 3$?
   A. $\frac{1}{2}, 1, 3$ B. -1, 1, 3 C. $-\frac{1}{2}, 1, 3$ D. $-3, \frac{1}{2}, 1$

6. Which polynomial function has zeroes at -4, 3, and 5?
   A. $f(x) = (x + 4)(x + 3)(x + 5)$
   B. $g(x) = (x + 4)(x - 3)(x - 5)$
   C. $h(x) = (x - 4)(x - 3)(x - 5)$
   D. $k(x) = (x - 4)(x + 3)(x + 5)$

7. Which is not a factor of $x^3 - x^2 - 17x - 15$?
   A. $x - 5$ B. $x + 1$ C. $x + 3$ D. $x + 5$

8. Which of the following is not a solution of $x^4 - 3x^2 - 54 = 0$?
   A. -3 B. 3 C. $-3i$ D. $-i\sqrt{6}$

9. What is the expanded form of $(a-b)^3$?

A. $a^3+a^2b+ab^2+b^3$

B. $a^3+3a^2b+3ab^2+b^3$

C. $a^3-a^2b+ab^2-b^3$

D. $a^3-3a^2b+3ab^2-b^3$

10. The function f is defined as $f(x)=6x^4+x^3+4x^2+x-2$.

- Using the Remainder Theorem, determine if $\frac{1}{2}$ is a root of $f(x)$. Explain.

- If i is also a root, what are the other two roots?

11. For a certain polynomial function, x = 3 is a zero with multiplicity of two and x = -3 is a zero with a multiplicity of one. Write a possible equation for this function and sketch its graph.

12. Is $(2x-3)^3-64$ equivalent to $(2x-11)(2x+5)$? Explain your reasoning.