The Ultimate Guide to Unit 7
Math 3 Honors

Name Key

Date: _____

1. A leprechaun is collecting gold coins for his coin collection. He has been collecting such coins every St. Patrick's Day since 1970. His collection has grown exponentially over the years and can be modeled by the function $f(t) = 50(1.04)^t$, where t is time in years and f(t) is number of coins in t years. Use this information to answer the following questions:

a. How many coins were in the collection when the collection began? 50

b. How many coins do we predict the leprechaun to collect on St. Patrick's Day 2016? 303-304

$50(1.04)^{46}$

c. In what year will the leprechaun collect his 10,000th coin? 2105 - 2106

$10000 = 50(1.04)^t$

$\log_{1.04} 200 = t$

2. Given $C(t) = 23(1.234)^t$, where t stands for the time in minutes and $C(t)$ stands for the length of a beanstalk in centimeters. Answer the following questions based on this information.

a. Give the height of the beanstalk after five minutes. $23(1.234)^5 = 65.81$

b. Give the height of the beanstalk after ½ hour. $23(1.234)^{30} = 12623.58$

c. How long would it take for the beanstalk to reach 1000 centimeters? $1000 = 23(1.234)^t$

d. How long was the beanstalk the instant it began to grow? 23 centimeters. $t = 17.94$ minutes.

3a. Solve for x: $\frac{1}{27} = 3^x$

$3^{-3} = 3^x$ $\boxed{x = -3}$

3b. Solve for x: $\frac{1}{2401} = 49^{x+1}$

$7^{-4} = 7^{2x+2}$

$-4 = 2x + 2$

$-6 = 2x$

$\boxed{x = -3}$

4. Suzie deposits $21 into an interest bearing savings account that yields continuous interest at a rate of 3.3%. How much money will she have in 18 years? When will her money quadruple?

$21e^{.033(18)} = \boxed{38.04}$

$e^{0.033(t)} = 4$

$\frac{\ln 4}{0.033} = \boxed{42 \text{ years.}}$

5. A tree is planted and starts growing continuously. Each month the height of the tree is measured. 3 months after the tree had been planted; it is about 11.849 meters tall. A year after the tree was planted; the tree is about 27.038 meters tall. What was the initial height of the tree?

$11.849e^{r(9)} = 27.038$

$e^{9r} = 2.28$

rate = 9.167%

$Pe^{.09167(12)} = 27.038$

$\boxed{9 \text{ meters}}$

6. The NHL begins the season with a population of 378 referees. Over the season due to bad calls and favoritism, the referees begin to get fired. After 63 games, there are about 200 referees still part of the league. What is the rate of decay for the population of referees?

$378(1-r)^{63} = 200$ 1%

7. Solve for the unknown variable in the equation: $\log_x 8 = 1.5$

$x^{1.5} = 8$ $x = 4$

8. State the domain of the logarithmic function: $\log_3 x = y$

D: $(0,\infty)$

9. Solve for the unknown variable in the equation: $4(2.3)^x = 48$

$12 = 2.3^x$ $\log_{2.3} 12 = x$ $x = 2.983$

10. Solve for the unknown variable in the equation: $49^{2x+2} = 343^{9x-4}$

-0.696 $7^{4x+4} = 7^{27x-12}$ $4x+4 = 27x-12$ $23x = -16$

11. State the inverse in terms of x of the function: $f(x) = x^3 + 9$

$x = y^3 + 9$ $\sqrt[3]{x-9} = f^{-1}(x)$

12. Find the domain for $f(x) = 321(0.5)^x$ $(-\infty, \infty)$

13. Find the range for $g(x) = 2.5(2)^x$ $(0,\infty)$

14. A. If the population of a city in 1923 is given by the equation $P(t) = 32021(1.03)^t$, what is the initial population? t stands for years since 1923.

32021 people.

b. What is rate of growth each year for the city? 3%.

c. How long will it take for the population to double? $2 = 1.03^t$ 23.45 years.

15. A car is purchased for $18,000 and depreciates in value 4.2% every month. How long will it take for the car to lose $8,000 in value? $10000 = 18000(1-0.042)^t$ $0.55 = 0.958^t$ 67 months

16. The growth of a sample of mold has been observed for the past 6 weeks. Three weeks ago, there were 28.1 grams of mold. Now there are 39.48 grams.

a.) What is the rate of growth? $28.1e^{r(3)} = 39.48$ 11.334%

b.) Find an equation to model the growth? $Pe^{0.11334(6)} = 39.48$ $20(e)^{0.11334t} = P(t)$

c.) What is the doubling time?

$2 = e^{0.11334t}$

$\frac{\ln 2}{0.11334}$ 6.116 weeks

15.432 years

$3 = 1.00375^{12t}$ = 24.459 years

17. An interest bearing savings account yields 4.5% annually, compounded monthly. When will your money double? Triple?

$2 = \left(1 + \frac{.045}{12}\right)^{12t}$ $\log_{1.00375} 2 = 12t$

18. $\log_4(x+1) + 3\log_2 4 = 8$ $\log_4 64x+64 = 8$ $65536 = 64x+64$ $x = 1023$

19. $4^{\log_7(x+2)} = \log_3 9$ $2^{\log_7(x+2)^2} = 2$ $\log_7 (x+2)^2 = 2$ $49 = x^2+4x+4$ $0 = x^2+4x-45$

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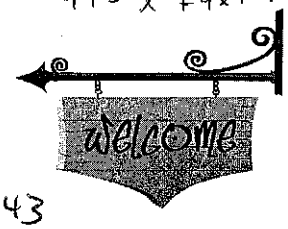
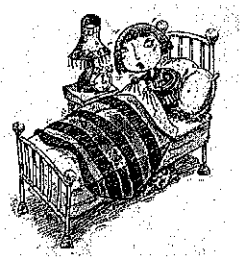
20. The population of a city is calculated every year. On January 1, 2012, the population of Maxwell Is Awesome, growing at about 3.6% yearly, is about 7643 people. What was the approximate population 12 years ago?



$P(1+0.036)^{12} = 7643$

about 5000



21. 60,000 little creatures are found underneath Kieran's bed. Too scared to kill them herself, she calls in Ryan to save the day. 15 minutes after his arrival, Ryan has killed 57,889 bugs. What is his rate of decay?

$60000(1-r)^{15} = 57889$

0.238%

22. A pile of leaves has been decaying for 6 days. Two days ago, there were 21 pounds of leaves left. Now 15 pounds are left.

a) What is the rate of decay? $21e^{k(2)} = 15$ -16.824%

b) Find an equation to model the decay. $Pe^{-0.16824(6)} = 15$ $P = 41.16e^{-0.16824t}$

c) What is the half-life of the pile of leaves? $\frac{1}{2} = e^{-0.16t}$ 4.12 days

23. Find the inverse of the function in terms of x: $f(x) = \sqrt{2x+1}$ and algebraically prove the two equations are inverses of each other.

$\sqrt{2\left(\frac{x^2-1}{2}\right)+1}$ $\frac{\left(\sqrt{2x+1}\right)^2 - 1}{2}$ $x = \sqrt{2y+1}$

$\sqrt{x^2-1+1}$ $\frac{2x+1-1}{2}$ $g(x) \frac{x^2-1}{2}$

$\sqrt{x^2}$ $\frac{2x}{2}$

x ✓ $y = \frac{2x}{2}$

24. Use properties of logarithms to solve for the variable given in each equation.

a) $4^x = 9$ $\log_4 9 = 1.585$

b) $\log_2 x = 9$ $2^9 = x$ 512

c) $6^{\log_6 x} = 9$ 9

d) $\log_4 2x + \log_2 7 = 9$ $\log_4 2x = 6.19$ 2674.94

e) $8.1^{\log_3 x} = 70$ $\log_{8.1} 70 = \log_3 x$ 9.311

f) $\ln x = \frac{2}{5}$ 1.492

g) $3\log_5 x - 2\log_5 x = \log_5 21$

$\log_5 x^3 - \log_5 x^2 = \log_5 21$ $\log_5 x = \log_5 21$ $x = 21$

h) $\log_4(3x-5) - \log_4(4x+2) = \frac{1}{5}\log_4 32 - \frac{1}{3}\log_4 27$

$\log_4 \frac{3x-5}{4x+2} = \log_4 \frac{2}{3}$ $3(3x-5) = 2(4x+2)$

$9x - 15 = 8x + 4$ $x = 19$

25. A team of scientists would like to know the initial height of Jack's beanstalk. They have determined that after 4.5 days, the beanstalk is 7,851 feet tall. They also have determined the stalk is growing continuously at 12.7%. Help the scientists determine the height of the beanstalk at day zero. $Pe^{.127(4.5)} = 7851$ $P = 4433.285$

26. Change each exponential function into a logarithmic function. Do Not Solve

a. $4^x = 12$ $\log_4 12 = x$ b. $\left(\frac{3}{4}\right)^x = 32$ $\log_{\frac{3}{4}} 32 = x$ c. $x^{-3} = 2$ $\log_x 2 = -3$

d. $7^{x+1} = 13$ $\log_7 13 = x+1$ e. $10 = 9^x$ $\log_9 10 = x$

27. Change each logarithmic function into an exponential function. Do Not Solve.

a. $\log_3 \frac{2}{3} = x$ $3^x = \frac{2}{3}$ b. $\log 10000 = x$ $10^x = 10000$ c. $\log_{\frac{1}{3}} x - 4$ $\left(\frac{1}{3}\right)^4 = x$

d. $\log_2 7 = 8$ $2^8 = 7$ e. $\log x = -3$ $10^{-3} = x$

28. Solve for *x*. Round to three decimal places. No more No less

a. $7^{3x-2} = 0.834$ 0.636 $\log_7 0.834 = 3x - 2$ b. $\log_2 6 + \log_2(x-2) = \log_2 3$

$\log_2(6x-12) = \log_2 3$

c. $\log x = \frac{1}{2}\log 81$ $x = 9$

$3 = 6x - 12$

$15 = 6x$

$\frac{15}{6} = x$

29. Humberto deposits $392.34 into a savings account that is compounded yearly at an annual rate of 2.45%. How long will it take Humberto to save enough money to buy a $1,243.14 flat-screen television? $392.34(1+0.0245)^t = 1243.14$ $\log_{1.0245} 3.169 = t$ 47.65 years

30. Find the inverse of each equation. Solve for y in each situation.

a. $y = -4x + 1$ $x = -4y + 1$ $\frac{x-1}{-4} = -\frac{1}{4}x + \frac{1}{4} = y^{-1}$

b $y = 2(4x+1)^2 - 1$ $x = 2(4y+1)^2 - 1$ $\frac{1}{4}\left(\sqrt{\frac{x+1}{2}} - 1\right)$

31. A hungry gorilla has found 900 pounds of bananas and begins to consume them at an amazing rate. 21 days later, the pile of bananas weighs only 100 pounds. Find the rate of decay for the bananas. $900e^{21(r)} = 100$ 10.461% decay

32. Solve for a: $2^{x+1} = 8$ $2^{x+1} = 2^3$ $x = 2$

33. Zoe deposits $100 into an account that yields 2.1% annual interest, compounded monthly. Find the amount of money she will have after 10 years. $100\left(1 + \frac{.021}{12}\right)^{10(12)} = \$123.35$

34. A population of baby elephants is growing at a continuous rate of 5.4% per year. Researchers discovered the current population to be at 3,783 elephants. Using this information, find the number of elephants that existed 8 years ago. $3783e^{-0.054(8)} \approx 2456$

35. What is the value of $\log_7 9$ rounded to 3 decimal places? 1.129

36. What makes the exponential function different from any other function?

variable in the exponent